Apparentment as a Matching Market: The Division of the Surplus Seat

Torun Dewan
Department of Government, London School of Economics
t.dewan@lse.ac.uk

Abstract

Under a divisor method of apportionment, two lists may sign an *apparentment*: they pool into one list for the allocation, and a seat the pool wins is split between them by the same method. In several systems, notably the Knesset, each list may sign with at most one partner. We study the resulting cooperative problem, whose coalitional worths are fixed by the apportionment law and so are computable, not assumed. Existing treatments regard an alliance as a single actor and leave open two questions a list faces: the *division* of the seat and the *matching* of who pairs with whom. The seat goes to the more under-represented partner, so *ex ante* it is split by recipiency probabilities. The single-partner rule makes formation a matching market. Our main result is a separation theorem: when a bloc is fully cohesive, both lists value a joint seat equally, so partner choice turns only on the pool's win probability, independent of the division, and a stable matching exists; as cohesion falls, the division enters partner choice and stability can fail. The agreements forgone, with the computable win and recipiency probabilities, place the cohesion weights in an identified set. We calibrate on five Knesset elections: the apportionment is reproduced exactly, the division rule holds in every capture, and in half the seat goes to the fewer-voted partner, which a proportional split cannot produce.

Keywords: apportionment; apparentment; surplus-vote agreements; d'Hondt method; matching; coalition formation; fair division.

JEL classification: D72; C78; D71.

ORCID: 0009-0007-8181-5584

1 Introduction

An *apparentment*, known also as a list alliance, an electoral cartel, or in Israel a surplus-vote agreement, lets two party lists be treated as one for the purpose of allocating seats. Their total votes are pooled, so the divisor method counts them as a single list, and the pair may win a seat that neither would have won apart. The device is routine under divisor methods of apportionment: it is used in German and Swiss local elections and, for the entire allocation, in elections to the Israeli Knesset (Bochsler, 2010; Leutgäb and Pukelsheim, 2009). It is also, for the analyst, an unusually clean object. The worth of every coalition of lists is fixed by the apportionment law and can be computed exactly from the vote returns, so the worths of the associated cooperative game are given rather than assumed, and the theory's predictions can be confronted with the agreements parties actually sign.[1]

The existing formal literature asks a single question: when is an apparentment worth signing. Karpov (2014) shows that under the d'Hondt method an alliance never loses seats, characterises the vote profiles under which no alliance helps anyone, and measures each party's a priori stake by the Shapley value of an alliance-versus-counteralliance game. Bochsler (2010) estimates the average gain empirically and, treating the alliance as a single actor, allocates it within the alliance in proportion to the members' vote shares. An older tradition predicts which alliances form, using the von Neumann–Morgenstern solution, with statistical support from the French National Assembly elections of 1951 (Lee et al., 1979). In each the alliance is a single player, and two questions a list faces once it contemplates signing are left open. If our pooled pair wins a seat, which of us gets it; and, because we may sign with only one partner, whom should we choose. The first is a problem of fair division of an indivisible prize, the second a problem of stable matching.

Three features carry the analysis beyond the single-actor view. First, the seat a pair wins is indivisible, and under the method it goes whole to the more under-represented partner, the one with the higher marginal average. This need not be the partner with more votes,[2] and it makes the *ex ante* division a split by recipiency probability rather than by vote share, contrary to the proportional rule that a reduced-form treatment assumes. Second, the single-partner rule turns formation into a *matching market*: a list chooses one partner from an admissible set, and the resulting profile is a matching. Third, the division and the matching interact through the cohesion of the bloc within which lists sign, because a seat that goes to a partner still stays in the bloc and is therefore worth something, though not everything, to a list that does not hold it.

[1] A pair's worth can in principle depend on the other agreements, making the game one of partition-function form; a separability condition, stated in Section 3, removes this dependence, and it is under that condition that the matching results are proved.

[2] The leading instance is the Knesset pair Shas and United Torah Judaism: Shas polled the more votes, yet the pooled surplus seat went to the smaller United Torah Judaism in 2022, September 2019, and April 2019.

Our main result, Theorem 1, is a separation theorem. When cohesion is complete, so that a list values a joint seat wherever it lands, the two partners value a pair equally, partner choice depends only on the pool's probability of winning a seat, and the division of the seat is irrelevant to who pairs with whom; moreover a stable matching then exists. As cohesion falls below one, and where the division is uneven, the two partners value the pair unequally, so the division enters each list's ranking of partners in proportion to the cohesion shortfall, and the game becomes an asymmetric roommate problem in which a stable matching may fail to exist. The matching and the division thus decouple exactly at full cohesion and couple as cohesion declines. A companion identification result shows that the agreements signed and forgone, together with the computable win and recipiency probabilities, set-identify the cohesion weights.

The paper's contribution is to bring two cooperative ideas, the fair division of a contested indivisible prize and stable matching, to bear on apportionment, where the primitives are supplied by law, and to state them for a general divisor method. Apportionment is a long-standing concern of this literature, from the axiomatic theory of Balinski and Young (2001) to the implementation of the Jefferson–d'Hondt rule of Pérez and De la Cruz (2014). Section 2 places the paper in that setting. Section 3 sets out the apparentment game for a general divisor method. Section 4 treats the division of the seat, Section 5 the matching and the separation theorem, and Section 6 gives a worked illustration. Section 7 calibrates the model on five Knesset elections, and Section 9 concludes. Proofs are collected in an appendix.

2 Related literature

The paper relates to three broad literatures. The first is the theory of apportionment. Divisor methods, their axiomatic characterisations, and their biases are the subject of Balinski and Young (2001) and Pukelsheim (2014); the d'Hondt method's property of never penalising a merger, which we invoke in Proposition 1, is the coalition-encouragement axiom that singles it out among divisor methods. The systematic bias of divisor methods toward larger lists, which apparentment lets small lists partly circumvent and whose net effect we take up in Section 8, is quantified by Schuster et al. (2003). Apparentment itself has been studied as a device that exploits the coalition-encouragement property: Leutgäb and Pukelsheim (2009) document its lottery-like effects in local elections, Bochsler (2010) estimates who gains, and Karpov (2014) settles when a successful alliance exists and measures the a priori gain by the Shapley value. Relative to these we take the alliance apart into its members and study the split and the pairing.

The second literature is fair division. The surplus seat is a contested indivisible prize, which places the division in the theory of claims problems, where an estate too small to meet all claims must be shared (O'Neill, 1982); the axiomatic and game-theoretic analysis of such problems is surveyed by Thomson (2003, 2015). Because the prize here is a single indivisible seat, the division also touches the literature on the fair division of indivisible items, in which two parties split a set of indivisible goods by a designed procedure (Brams and Kilgour, 2001; Brams et al., 2012). The Talmud's two-claimant contested-garment rule, and its consistent extension to n claimants, coincide with the nucleolus of the associated coalitional game (Aumann and Maschler, 1985; Schmeidler, 1969). The same contested-garment logic has been used to model how a governing coalition divides office among its partners (Dewan, 2026). We read that correspondence in the other direction: the surplus seat is a two-list contested garment, the "claim" is the marginal average, and the *ex ante* award tracks it. Unlike the designed procedures of that literature, and unlike the textbook claims problem, here the estate is a single indivisible seat and the divider is the apportionment law itself, which awards the seat by a priority rule rather than by contract or by a mechanism the parties choose.

The third literature is matching. Because each list may sign with at most one partner, formation is akin to a roommate problem (Gale and Shapley, 1962), the one-sided counterpart of the marriage market. Unlike the marriage market, a roommate problem need not admit a stable matching (Gale and Shapley, 1962): Irving (1985) gives an algorithm that returns one or reports that none exists, and Tan (1991) characterises existence exactly, a stable matching existing if and only if no stable partition contains an odd ring, with a related sufficient no-odd-rings condition due to Chung (2000). We put this apparatus to work in Section 5: the separation theorem shows that full cohesion gives each pair a common value to both lists, which forbids rings and so restores existence, and the odd-ring characterisation locates the exact boundary once cohesion falls. The broader theory of one-sided and two-sided matching is surveyed by Roth and Sotomayor (1990).

The paper also speaks to the study of electoral alliances. Predicting which alliances form by a cooperative solution concept goes back to Lee et al. (1979), on the French elections of 1951; Blais and Indridason (2007) give the logic of alliances in two-round legislative elections, and the empirical determinants of pre-electoral coalitions, ideological proximity and the size of the prospective bloc, are studied by Golder (2006). Closest to our setting, Hortalà-Vallvé et al. (2024) examine joint lists under proportional representation, where allied parties pool onto a single list as here; they find such coalitions form between parties of similar size and ideology and reshape the distribution of seats, a counterpart to our matching and disproportionality results. We differ from these in taking the coalitional worths as fixed by the apportionment law and hence computable, and in the one-partner matching formulation, in which the division of the seat feeds back into who pairs with whom.

3 The apparentment game

Divisor methods. A set of lists $N = \{1, \dots, n\}$ with vote totals $v = (v_1, \dots, v_n)$, $v_i > 0$, competes for S seats. A *divisor method* is given by a strictly increasing sequence $0 < d(0) < d(1) < \cdots$; seats are awarded one at a time to the list of highest *marginal average*

$$a_i(s_i) = \frac{v_i}{d(s_i)},$$

where s_i is the number of seats i already holds. The d'Hondt (Jefferson) method is $d(s) = s+1$ and Sainte-Laguë is $d(s) = 2s+1$. Write s^0 for the apportionment without apparentments and $a_i = a_i(s_i^0)$ for the resulting marginal averages. We assume no ties, which holds for generic v.

Apparentments and the game. An *apparentment profile* is a matching M on N, a set of disjoint pairs with each list in at most one, drawn from an admissible graph G that in applications collects the pairs within a common ideological bloc. Given M, each pair $\{i, j\} \in M$ is treated as a single list with the pooled votes $v_i + v_j$; all S seats are then recomputed among the pooled and unpooled lists, and each pool's aggregate is divided between its members by the same divisor method. The seat counts $s_i^0 + s_j^0$ are not an endowment but a benchmark, the pool's no-agreement total, against which its gain is measured; Definition 1 states the map in full. Let $A_i(M)$ denote i's total seats under M. For an admissible pair, its *gain* is

$$g_{ij}(M) = \big[A_i(M + ij) + A_j(M + ij)\big] - \big[A_i(M) + A_j(M)\big],$$

the extra seats the two win together rather than apart, holding the rest of M fixed. The pair (N, G) together with the map A is the *apparentment game*; it is determined by v and the method.

Definition 1 (The allocation map)**.** Given a profile M, replace each pair $\{i, j\} \in M$ by a single list of $v_i + v_j$ votes to form the reduced list set; apply the divisor method to award all S seats among the reduced lists; and split each pooled list's awarded seats between its two members by the same divisor method applied to (v_i, v_j). The result is $A(M) = (A_1(M), \dots, A_n(M))$, defined for every simultaneous matching M. For d'Hondt this is exactly the Bader–Ofer implementation used in Section 7, where the reconstruction reproduces each official allocation, confirming the equivalence.

Proposition 1 (Non-negativity under d'Hondt)**.** *Under the d'Hondt method $g_{ij}(M) \geq 0$ for every admissible pair and profile: an apparentment never costs its members a seat. This property fails for a general divisor method.*

That d'Hondt is the unique divisor method encouraging coalitions in this sense is a result due to Balinski and Young (2001); the profile-level statement is due to Karpov (2014). It is why apparentment is paired with d'Hondt in practice, and we take $g_{ij} \in \{0,1\}$, a pair pivotal for at most one seat, as the leading case. The division results of Section 4 hold for every divisor method; the matching results of Section 5 are stated for d'Hondt under the separability condition below, which makes the pooling of a pair independent of the rest of the profile.

Assumption 1 (Separable blocs)**.** The admissible graph G partitions into blocs that contest disjoint remainder seats, with at most one pivotal pair per bloc. Under separability, forming one agreement leaves every other pair's outcome unchanged, so the gain g_{ij}, the win probability $p_{ij} = \Pr[g_{ij} = 1]$, and the recipiency $q_{ij} = \Pr[a_i > a_j \mid g_{ij} = 1]$ are matching-independent, evaluated against the fixed no-agreement background s^0; the marginal averages in Proposition 2 are then those of s^0.

Separability is what turns the profile-dependent $g_{ij}(M)$ of the game into the fixed win and recipiency probabilities from which the matching utilities are built, and it holds exactly when blocs compete for disjoint seats, the case in which apparentment is normally deployed. The fully profile-dependent problem, in which forming one pair shifts another pair's quotients, is a matching game with externalities, taken up at the end of Section 5; the empirical probabilities of Section 7 are computed under the same fixed-background convention, holding the other blocs' agreements at their observed values, and Table 2 shows the estimates are stable across the perturbation scale.

Remark 1 (Method dependence)*.* Proposition 1 is special to d'Hondt. Under Sainte-Laguë, $d(s) = 2s + 1$, a merger can lose a seat. With three lists of votes $(51, 25, 24)$ and $S = 3$, Sainte-Laguë awards one seat to each list; pooling the two smaller lists gives two seats to the largest and one to the pool, so the pooled pair falls from two seats to one. Apparentment is thus a free option only under a coalition-encouraging method, which is why the systems that permit it use d'Hondt. The division rule of the next section, by contrast, holds under every divisor method.

4 Division of the seat

Fix a pair with $g_{ij} = 1$: pooling wins the two lists one seat they would not have won apart.

Proposition 2 (The seat goes to the more under-represented partner)**.** *Under any divisor method the seat is assigned to the member with the higher marginal average, i if $a_i > a_j$.* Ex post *this is a priority rule.* Ex ante*, before votes are realised, the pool's expected seat is split,*

$$\Big(\Pr[a_i > a_j \mid g_{ij} = 1],\ \Pr[a_j > a_i \mid g_{ij} = 1] \Big),$$

each partner's share being its probability of being the recipient.

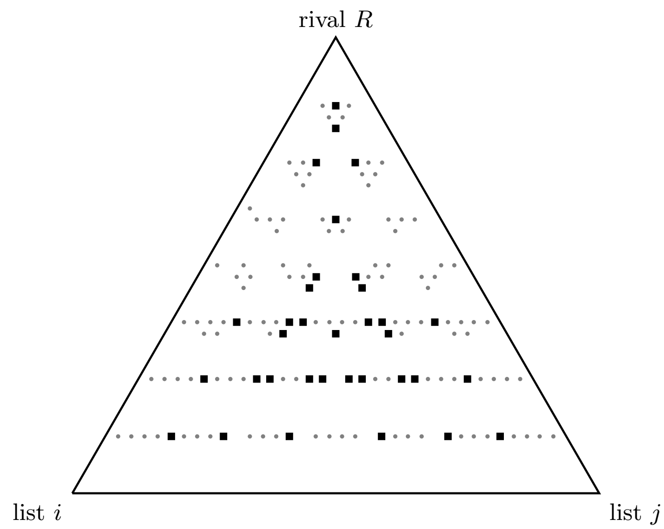

Figure 1: Where an apparentment succeeds and who wins the seat. The simplex of vote shares for two bloc lists i, j and a rival R under d'Hondt with $S = 7$. Each point in the triangle is a profile of the three lists' vote shares, which sum to one; a list's share rises as the point moves toward its labelled corner. In the blank region the pool wins no extra seat. Grey points: the pool wins a seat, taken by the larger list. Black squares: the pool wins a seat, taken by the smaller list, the reversal region of Proposition 3.

The higher marginal average is the list with the larger vote-per-prospective-seat and so the more under-represented at the margin. Because a list's average falls as it accumulates seats, this need not be the list with more votes: a larger list already holding more seats can have the lower average and cede the seat to a smaller partner. The *ex ante* object is then a genuine division of a contested indivisible prize, with the marginal average in the role of the claim (O'Neill, 1982; Aumann and Maschler, 1985), and it departs from the proportional split of Bochsler (2010) precisely when votes and marginal averages disagree, an event we find common in the data.

Proposition 3 (When the fewer-voted partner wins)**.** *Order the pair so that $v_i > v_j$. The seat goes to the fewer-voted list j if and only if its marginal average is the higher,*

$$\frac{v_j}{d(s_j^0)} > \frac{v_i}{d(s_i^0)}, \qquad \text{and for d'Hondt} \qquad s_j^0 + 1 < \frac{v_j}{v_i}\,(s_i^0 + 1).$$

Equivalently, under d'Hondt the smaller list wins exactly when it trails the larger in seats by more than its vote deficit, $s_i^0 - s_j^0 > \left(1 - \frac{v_j}{v_i}\right)(s_i^0 + 1)$. A proportional split never produces such a reversal, so the frequency of reversals measures the divergence between the two rules; in the five elections below it is one half of the captures.

Figure 1 shows the reversal region for a bloc of two lists against an outside rival.

5 Matching, and the separation theorem

Throughout this section we impose separability (Assumption 1), so that p_{ij} and q_{ij} are exogenous. A seat that goes to a partner still stays in the bloc, so a list values a joint seat held by its partner at a weight $\eta_{ij} \in [0, 1]$, the *cohesion* of the pair, high within a tight bloc and low across a loose one.[3] We take cohesion to be symmetric, $\eta_{ij} = \eta_{ji}$: it is a property of the two lists jointly, both valuing a bloc seat held by the other equally. (Asymmetric cohesion adds a term $p_{ij}[\eta_{ij}(1 - q_{ij}) - \eta_{ji}(1 - q_{ji})]$ to the difference below and does not change the qualitative results.) Let $p_{ij} = \Pr[g_{ij} = 1]$ be the pool's win probability and $q_{ij} = \Pr[a_i > a_j \mid g_{ij} = 1]$ list i's recipiency probability, with $q_{ji} = 1 - q_{ij}$. List i's value from signing with j is

$$u_i(j) = p_{ij}\big[q_{ij} + \eta_{ij}\,(1 - q_{ij})\big],$$

its chance of the pool winning times its own recipiency plus the cohesion-weighted chance the partner takes the seat; being unmatched is worth 0. We call p_{ij} the *edge value* of the pair, a single number attached to the undirected pair and symmetric in its two lists. The utility $u_i(j)$ is not in general an edge value: unless $\eta_{ij} = 1$ the two lists value the pair differently, $u_i(j) \neq u_j(i)$, so the pair carries a preference profile rather than a scalar. At full cohesion the two utilities collapse to the edge value, $u_i(j) = u_j(i) = p_{ij}$, and it is that case in which the problem reduces to a weighted matching.

Definition 2 (Pairwise stability)**.** A matching M on G is *stable* if every matched list weakly prefers its partner to being unmatched, and there is no admissible $\{i, j\} \notin M$ with $u_i(j) > u_i(M)$ and $u_j(i) > u_j(M)$, where $u_i(M)$ is the value of i's pair in M, or 0 if i is unmatched.

Theorem 1 (Separation, existence, and coupling)**.** *Consider the one-partner apparentment game with cohesion weights $\eta_{ij} \in [0, 1]$.*

(i) Separation. *If $\eta_{ij} = 1$ then $u_i(j) = u_j(i) = p_{ij}$: both partners value the pair by the pool's win probability alone, and partner choice is independent of the division q_{ij}.*

(ii) Existence. *If $\eta_{ij} = 1$ for all admissible pairs, a pairwise-stable matching exists; the greedy matching that repeatedly adds the highest-p admissible edge between two currently unmatched lists is stable.*

(iii) Coupling. *For $\eta_{ij} < 1$, $u_i(j) - u_j(i) = p_{ij}(1 - \eta_{ij})(q_{ij} - q_{ji})$, so the partners value the pair unequally when the division is uneven, the recipiency q_{ij} enters i's ranking with weight $(1 - \eta_{ij})$, and a stable matching may fail to exist.*

Part (i) is the organising point: a fully cohesive bloc reduces partner choice to maximising the pool's chance of a seat, and who ends up holding the seat is then a purely distributional

[3] In the Knesset these tight blocs are the two *haredi* lists, Shas and United Torah Judaism, and the two nationalist lists, Likud and Religious Zionism; lists sign within bloc (Section 7).

question. Part (ii) turns the common valuation of each edge into existence by a greedy argument. Part (iii) shows that the division and the matching couple exactly as cohesion falls below one, and that below full cohesion the game is an asymmetric roommate problem; Propositions 4 and 5 then give a sufficient condition and the exact condition for a stable matching to exist.

Proposition 4 (Existence under a common order)**.** *Suppose there is a strict order $\succ$ on the admissible pairs such that every list ranks its admissible partners in agreement with $\succ$: $u_i(j) > u_i(k)$ whenever $\{i,j\} \succ \{i,k\}$. Then the greedy matching that adds pairs in $\succ$-order between currently unmatched lists is pairwise stable, so a stable matching exists. Full cohesion is the special case $\succ=$ the order by p (Theorem 1(ii)), and the non-existence instance of Theorem 1(iii) is precisely a profile that admits no such common order.*

Proposition 4 gives a sufficient condition spanning the cases of Theorem 1: stability is guaranteed whenever the induced partner rankings are consistent with a single order of pairs, a property that holds at full cohesion and can fail once cohesion and division pull the rankings into a cycle, as in Theorem 1(iii). The common order is sufficient but not necessary, and the exact boundary is known.

In order to state the boundary we adopt the parlance of the roommate problem. Two structures matter: a cyclic pattern of preferences, the ring, and the bookkeeping device that always exists, the stable partition.

Definition 3 (Ring and odd ring)**.** In the roommate problem induced on G by the utilities $u_i(\cdot)$, in which each list ranks its admissible partners above remaining unmatched, a *ring* is an ordered set of lists $(x_1, \ldots, x_k)$ with $k \geq 3$, consecutively admissible around a cycle, in which each list prefers its successor to its predecessor:

$$u_{x_t}(x_{t+1}) > u_{x_t}(x_{t-1}) \qquad \text{for every } t \quad \text{(indices modulo } k\text{)}.$$

The ring is *odd* if k is odd. A *stable partition* (Tan, 1991) is a partition of the lists into pairs, singletons, and rings against which no pair blocks; every roommate problem admits one.

Proposition 5 (Exact existence)**.** *Take the induced preferences to be strict, as they are for generic votes and cohesion. Under separability the apparentment game is a roommate problem on G, and by Tan (1991) it has a pairwise-stable matching if and only if some, equivalently every, stable partition contains no odd ring. The boundary then has two transparent faces. A common order (Proposition 4) forbids a ring of any length, so it is sufficient for existence and contains full cohesion, where the order by p prevails, as a special case. Existence fails only through an odd ring, and the smallest is the three-list cycle $1 \to 2 \to 3 \to 1$ of Theorem 1(iii), the least configuration in which cohesion and a cyclic recipiency defeat every matching.*

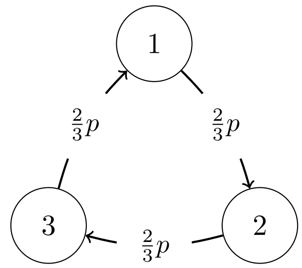

Figure 2: The odd ring of Proposition 5, on the three-list cycle of Theorem 1(iii). An arrow runs from each list to the partner it prefers, worth $\frac{2}{3}p$ against $\frac{1}{3}p$ for the other. Any single pairing leaves one list unmatched, and the arrow into that list marks the pair that blocks it: $\{1,2\}$ strands 3, and $\{2,3\}$ then blocks, so no matching is stable. An even ring, by contrast, pairs off and is harmless.

A ring is a chain of lists each chasing its successor. Figure 2 draws the smallest one, the three-list cycle of Theorem 1(iii): with $\eta = 0$ and cyclic recipiency $q_{12} = q_{23} = q_{31} = \frac{2}{3}$, each list values its successor at $\frac{2}{3}p$ and its predecessor at $\frac{1}{3}p$, so every list would rather switch forward. Any single pairing then leaves one list unmatched, and the arrow into that list marks the pair that blocks: matching $\{1,2\}$ strands 3, whom 2 would rather join, and the same holds around the cycle, so no matching survives. Around an even ring, by contrast, the lists pair off two by two and the cycle is harmless; only an odd ring, which always leaves one list over, obstructs. Because a ring needs at least three lists, this three-list cycle is the smallest obstruction, which is why the coupling of Theorem 1(iii) first bites at three lists and not before. The no-odd-ring condition is thus exact: necessary and sufficient, with the common order a clean sufficient case and full cohesion, which induces the order by p, inside it.

Corollary 1 (Stability need not be efficient)**.** *Even under full cohesion, a stable matching need not maximise the bloc's total win probability $\sum_{\{i,j\}\in M} p_{ij}$: stability and efficiency can conflict.*

The tension is standard for matching, but here it has real consequences, since the "efficient" matching is the one that wins the bloc the most seats. A high-p pair can pre-empt two moderate pairs whose combined win probability is larger, yet the two moderate pairs are unstable because the high-p pair blocks them (Figure 3).

Proposition 6 (The identified set of cohesion weights)**.** *Fix the admissible graph G and the computable (p_{ij}, q_{ij}). A signed pair constrains its own cohesion only by $\eta_{ij} \geq 0$ and so is uninformative; identification comes from the pairs not signed. For each admissible $\{i,j\} \notin M$, pairwise stability requires that $\{i,j\}$ not block the observed matching,*

$$u_i(j) \leq u_i(M) \quad \textit{or} \quad u_j(i) \leq u_j(M),$$

a disjunction in the cohesion weights of the members' realised pairs. The identified set is the intersection, over unsigned admissible pairs, of these unions of two half-spaces, in general a union of regions rather than an edge-by-edge box.

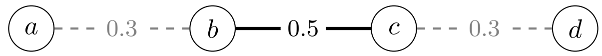

Figure 3: Stability versus efficiency (Corollary 1). Admissible pairs are edges labelled by the pool's win probability p. The stable (greedy) matching is the single heavy edge $\{b, c\}$, total 0.5; the efficient matching is the two dashed edges $\{a, b\}, \{c, d\}$, total 0.6, but it is blocked by $\{b, c\}$ and so is not stable.

Signing is uninformative because individual rationality is automatic: $u_i(j) \geq 0$ for every $\eta \geq 0$, so a chosen pair imposes nothing beyond its own non-negativity. Because cohesion is pair-specific and only one matching is observed per election, point or interval identification requires the explicit graph G together with cross-edge restrictions, for example a common cohesion function of ideological distance, or pooling across elections.

The profile-dependent game

Assumption 1 treats the probabilities as exogenous. Dropping it, a list's value from a pair depends on the whole profile, $u_i(j; M) = p_{ij}(M)\,[q_{ij}(M) + \eta_{ij}(1 - q_{ij}(M))]$, through the profile-dependent gain $g_{ij}(M)$. A deviation to $\{i, j\}$ dissolves i's and j's current pairs, and $\{i, j\}$ *blocks* M if both strictly prefer the induced profile $M' = M \setminus \{M(i), M(j)\} \cup \{ij\}$, that is $u_i(j; M') > u_i(M)$ and $u_j(i; M') > u_j(M)$. Under separability the probabilities are constant in M and this is exactly the game of Section 5, so Theorem 1 and Proposition 5 apply.

The economic content of the externality is that agreements are typically substitutes: a seat one pool captures is one fewer among the scarce marginal seats the others contest, so forming a pair tends to lower the others' gains. The leave-one-out reconstruction bears this out; in 2022, given the Likud agreement, the UTJ–Shas agreement is redundant, capturing no marginal seat. When the gains are monotone in this way the separable analysis is a good approximation. But monotonicity is not general: under d'Hondt the interaction of several alliances is the "lottery effect" of Leutgäb and Pukelsheim (2009) and Karpov (2014), in which pooling can raise as well as lower a rival pool's total, so a pair's value is neither monotone nor separable in the rest of the profile; and matching problems with externalities of this kind need not possess a stable outcome (Sasaki and Toda, 1996). We therefore delimit rather than resolve the general game: the fixed-background theory of Section 5 holds exactly under separability and approximately under substitutability, the empirically relevant cases, while the fully general lottery interaction is left open.

6 A numerical illustration

Take four lists with votes $v = (v_R, v_1, v_2, v_3) = (64, 37, 23, 31)$ in thousands, a house of $S = 10$, and the d'Hondt method, where R is a rival outside the bloc $\{1, 2, 3\}$. The apportionment without apparentments is $s^0 = (5, 2, 1, 2)$, and the marginal averages are $a_1 = 12.33 > a_2 = 11.50 > a_R = 10.67 > a_3 = 10.33$. List 3 has more votes than list 2 yet a lower marginal average, because it already holds an extra seat. Each within-bloc pair gains the bloc one seat, and by Proposition 2 the recipient is the higher-average member: $\{1,2\} \to 1$, $\{1,3\} \to 1$, $\{2,3\} \to 2$. In $\{2,3\}$ the seat goes to the fewer-voted list 2. This is a realised, deterministic configuration, in which all three pairs gain a seat, so the win probabilities are degenerate at one and the matching turns on the division and on cohesion, not on p. With one partner per list only one pair forms, and list 3, the lowest average, is the donor in any pair it joins, never the recipient. The pairwise-stable matchings are $\{1,3\}$, which gives the seat to 1, and $\{2,3\}$, which gives it to 2; list 3 is the donor in both, and lists 1 and 2 compete to secure it. Lowering the cohesion of $\{2,3\}$ alone does not forfeit the bloc's seat, since $\{1,2\}$ and $\{1,3\}$ remain available; it removes one of the two stable matchings. The role of the win probability, and a genuine choice among pairs by p, appears only once votes are uncertain, as in the calibration of Section 7.

A larger bloc shows the coupling of Theorem 1(iii). Take a rival R with 520 thousand votes and a bloc of four lists with votes $(91, 77, 60, 44)$, and $S = 12$ under d'Hondt. The apportionment without apparentments gives R nine seats and the bloc $(1, 1, 1, 0)$, with marginal averages 52.0 for R and $(45.5, 38.5, 30.0, 44.0)$ for the bloc. List 4, the fewest-voted but seatless, has the second highest average in the bloc and is badly under-represented. Two pairs gain a seat: $\{1,4\}$, with the seat going to list 1, and $\{2,4\}$, with the seat going to the fewer-voted list 4, a reversal in the sense of Proposition 3. List 4 lies in both gaining pairs and is pivotal. Under full cohesion it is indifferent between them, both are stable, and either seat stays in the bloc. Once cohesion is imperfect, list 4 strictly prefers $\{2,4\}$, in which it holds the seat, to $\{1,4\}$, in which it is the donor; the unique stable matching is then $\{2,4\}$, and list 1, which could have won the seat through $\{1,4\}$, is shut out. The division has selected the matching, exactly as Theorem 1(iii) describes.

7 Calibration: five Knesset elections

The realised objects, the votes, the allocation, the marginal averages, and the recipients, are observed directly from the returns; the win and recipiency probabilities are simulated under an assumed pre-election vote distribution, and the cohesion weights remain latent, set-identified as in Proposition 6. We reconstruct the Bader–Ofer (d'Hondt) allocation for the five elections from April 2019 to 2022 from the official vote returns and the surplus-vote agreements the parties signed, dropping agreements with a below-threshold partner as void.

Table 1 reports the exact result. The reconstruction is exact in all five elections, and the division rule holds in every one of the eight seat-captures. Because that rule is the method's own internal allocation, the 8/8 is a check on the reconstruction and on Proposition 2, not an independent behavioural test. The behavioural content lies elsewhere: in half of the captures the seat goes to the fewer-voted partner, the reversal of Proposition 3 that a proportional split

Table 1: Surplus-vote agreements and the division rule.

Election	Parties	Agreements	Capturing pairs	Higher-average wins	Seat to smaller party
April 2019	11	4	1	1/1	1
September 2019	9	4	1	1/1	1
2020	8	3	1	1/1	0
2021	13	5	3	3/3	1
2022	10	3	2	2/2	1
Total		19	8	8/8	4

Notes. Qualifying parties are those above the 3.25% threshold. "Capturing pairs" are agreements that win their pool an extra seat against the no-agreement baseline. The reconstruction reproduces each official 120-seat allocation exactly. In every capture the seat goes to the higher-marginal-average member (Proposition 2); in four of eight it goes to the partner with fewer votes (UTJ over Shas in 2022, September 2019 and April 2019; Meretz over Labor in 2021), which a proportional split cannot produce.

Table 2: The 2022 within-bloc admissible graph: win probability and recipiency split.

Bloc	Pair	p (win a seat)	recipiency split
Right	Likud + Religious Zionism*	0.31	0.69 : 0.31
	Likud + Shas	0.32	0.75 : 0.25
	Likud + UTJ	0.34	0.80 : 0.20
	Religious Zionism + Shas	0.39	0.56 : 0.44
	Religious Zionism + UTJ	0.41	0.65 : 0.35
	Shas + UTJ*	0.42	0.58 : 0.42
Centre	Yesh Atid + National Unity*	0.35	0.67 : 0.33
	Yesh Atid + Yisrael Beiteinu	0.37	0.80 : 0.20
	Yesh Atid + Labor	0.37	0.82 : 0.18
	National Unity + Yisrael Beiteinu	0.42	0.66 : 0.34
	National Unity + Labor	0.42	0.70 : 0.30
	Yisrael Beiteinu + Labor	0.44	0.53 : 0.47
Arab	Ra'am + Hadash–Ta'al	0.43	0.51 : 0.49

Notes. All admissible within-bloc pairs for 2022; * marks the agreements actually signed. Vote shares perturbed by Gaussian noise of standard deviation 1.5 points, 20,000 draws, with the threshold applied in each draw. The win probabilities are stable across the perturbation scale: for the signed pairs p moves by at most 0.01 as the noise ranges over 1.0 to 2.0 points. The 2021 graph is analogous and appears in the replication file.

cannot produce; and in the matching of who signs with whom, taken up next. The full profile of agreements moves at most one seat per election and in either direction: within the right in April 2019, from the right to Meretz in 2021, and from Labor to the right in 2022.

Ex ante behaviour. Agreements are signed before the votes, so we perturb each party's share by polling-scale noise and re-run the allocation to obtain the win probability p and recipiency split q of every admissible within-bloc pair, signed or not (Table 2); stability and cohesion recovery require the alternatives, not only the chosen pairs.

No signed agreement is a sure thing: every win probability lies between 0.30 and 0.44. Because an apparentment never loses a seat (Proposition 1), signing is a free option and parties take the best available, which is why many agreements capture nothing *ex post*. The recipiency is genuinely split, not degenerate, as with Shas and UTJ at 0.58 and 0.42. The matching is not explained by the arithmetic: within each bloc the signed pairs are not those of highest win probability. The pattern is consistent with graded rather than binary cohesion. A coarse partition would over-predict: Yisrael Beiteinu and Labor carry the centre bloc's highest win probability, $p = 0.44$ in Table 2, yet go unsigned, which is hard to reconcile with equal within-bloc cohesion between two lists a crude bloc dummy would treat as identical; and within the right bloc, where every perfect matching yields nearly the same total win probability, the signed pairs are the natural affinities, the two *haredi* lists (Shas and UTJ) and the two nationalist lists (Likud and Religious Zionism). We stop short of establishing graded cohesion: by Proposition 6 the signed pairs, the unsigned alternatives, and the admissible graph together place the cohesion weights in an identified set, and formally ruling out binary cohesion would require solving that joint set, which we leave to estimation on a longer panel.[4]

8 Apparentment and disproportionality

Does letting small lists coordinate offset the large-list bias of the d'Hondt method, or deepen it? The answer is neither in general. An apparentment moves one remainder seat from the list ℓ that would otherwise have held the last seat to the pool's recipient r, the more under-represented member of the winning pair. Whether this raises or lowers a least-squares index of disproportionality depends on the representation of r relative to ℓ, which the pooling does not fix: the pool out-averages ℓ on combined votes, but its recipient's own marginal average may lie on either side of ℓ's.

Proposition 7 (Ambiguous effect on disproportionality)**.** *Apparentment changes a least-squares index of disproportionality by transferring one seat from the displaced list ℓ to the recipient r. Writing e_k for list k's seat-share-minus-vote-share deviation and $\delta = 1/S$ for one seat's share, the transfer changes the sum of squared deviations by $2\delta(e_r - e_\ell + \delta)$. The index falls, becoming less disproportional, if and only if $e_\ell - e_r > \delta$: the displaced list must be more over-represented than the recipient by more than a single seat's share. This need not hold, so apparentment has no systematic direction.*

Table 3 bears this out on the Knesset data. The agreements lowered disproportionality in two of the five elections, raised it in one, and left it unchanged in the two where no seat moved. The sign tracks the seat: in 2021 it passed from an over-represented Likud to an under-represented Meretz and the index fell by nearly half a point; in 2022 it passed from Labor to an already over-represented Likud and the index rose. Coordination among smaller lists is thus not a corrective to the method's bias as such; it corrects the bias only when the

[4] Such estimation would recover the cohesion weights by revealed preference from the identified set of Proposition 6, and could pool across the many divisor-method systems that permit list linkage.

Table 3: Gallagher disproportionality, with and without agreements.

Election	Without agreements	With agreements	Change
April 2019	1.16	0.86	−0.30
September 2019	0.98	0.98	0.00
2020	0.65	0.65	0.00
2021	1.15	0.74	−0.41
2022	0.65	1.08	+0.43

Notes. Gallagher least-squares index over qualifying parties, seat shares of 120 against vote shares, computed from the reconstructed allocations with and without the signed agreements.

coordinating recipient is the more under-represented of the two lists whose fortunes the seat decides.[5]

9 Concluding remarks

Apparentment turns an apportionment technicality into a small, exact cooperative problem: a contested seat manufactured by pooling, divided by a priority the method specifies, among partners chosen in a one-to-one market. The division and the matching are independent problems when the bloc is fully cohesive, since utilities are then insensitive to recipiency; they couple only as cohesion falls, when the division re-enters the choice of partner. Because the primitives are legislated, the account is testable, and the calibration confirms the division rule while showing that partner choice follows a graded cohesion a bloc dummy misses.

The reversal characterisation of Proposition 3 and Figure 1 identify the region of the vote space in which the fewer-voted list captures the seat, and the calibration shows that region is reached in half of the observed captures, United Torah Judaism over the larger Shas among them; this is the sharpest way in which the divisor-method priority departs from a proportional split, and it concerns the analyst rather than the parties. The parties' own problem is the matching, which Theorem 1 shows separates from the division only under full cohesion; the graded cohesion we recover in the data places the applications squarely in the coupled regime, where who signs with whom turns on who would hold the seat. On the normative side, Proposition 7 and Table 3 caution against reading apparentment as a device that tames the method's large-list bias: it moves a single seat whose distributive sign is set by the configuration, and in the Knesset it has gone both ways.

By taking the alliance apart into its members, the paper turns a counting rule into a testable theory of how a surplus seat is divided and which lists combine to win it, with every primitive supplied by the apportionment law.

[5] A systematic welfare comparison of divisor methods under apparentment, in the axiomatic tradition of Balinski and Young (2001), is a natural next step.

Appendix: Proofs

Proof of Proposition 1. That the d'Hondt (Jefferson) method never penalises a merger, so a pool wins at least the sum of its members' separate seat counts, is its coalition-encouragement property, established by Balinski and Young (2001) and stated at the profile level by Karpov (2014); hence $g_{ij} \geq 0$. The property is not a consequence of pointwise domination of the two lists' quotient sequences but of the divisor sequence being $d(s) = s + 1$, and we take it from those references rather than reproduce it. That a non-d'Hondt divisor method can make a merger lose a seat is the corresponding failure of coalition encouragement (Balinski and Young, 2001), illustrated in Remark 1. □

Proof of Proposition 2. When the pooled pair receives a seat, the internal application of the divisor method awards the next seat to the list of highest marginal average among the two, namely $\arg\max\{a_i, a_j\}$; with $a_i > a_j$ this is i. Taking expectations over the vote distribution at the time of signing gives the stated split, the two shares summing to one on the event $g_{ij} = 1$. □

Proof of Proposition 3. By Proposition 2 the recipient is $\arg\max\{a_i, a_j\}$; with $v_i > v_j$ the seat goes to j iff $a_j > a_i$, i.e. $v_j/d(s_j^0) > v_i/d(s_i^0)$. For d'Hondt, $d(s) = s + 1$, cross-multiplying gives $v_j(s_i^0 + 1) > v_i(s_j^0 + 1)$, i.e. $s_j^0 + 1 < (v_j/v_i)(s_i^0 + 1)$. Subtracting both sides from $s_i^0 + 1$ yields $s_i^0 - s_j^0 > (1 - v_j/v_i)(s_i^0 + 1)$. □

Proof of Theorem 1. (i) Setting $\eta_{ij} = 1$ in $u_i(j) = p_{ij}[q_{ij} + \eta_{ij}(1 - q_{ij})]$ gives $u_i(j) = p_{ij}(q_{ij} + 1 - q_{ij}) = p_{ij}$, and symmetrically $u_j(i) = p_{ij}$; neither involves q_{ij}.

(ii) With $\eta \equiv 1$ each admissible edge $\{i, j\}$ carries the common value p_{ij} to both endpoints. Order the admissible edges by p in decreasing order and process them, adding an edge to M whenever both endpoints are still unmatched. Suppose the resulting M has a blocking pair $\{i, j\}$, so $p_{ij} > u_i(M)$ and $p_{ij} > u_j(M)$. When $\{i, j\}$ was processed, if either endpoint were already matched it would be to an edge of value at least p_{ij}, processed earlier, contradicting $p_{ij} > u_i(M)$ or $p_{ij} > u_j(M)$; hence both endpoints were unmatched and the procedure would have added $\{i, j\}$, a contradiction. Thus M is stable.

(iii) Directly, using $q_{ji} = 1 - q_{ij}$,

$$u_i(j) - u_j(i) = p_{ij}\big[(q_{ij} + \eta_{ij}(1 - q_{ij})) - (q_{ji} + \eta_{ij}(1 - q_{ji}))\big] = p_{ij}(1 - \eta_{ij})(q_{ij} - q_{ji}),$$

which is nonzero whenever $\eta_{ij} < 1$ and $q_{ij} \neq \frac{1}{2}$; and $\partial u_i(j)/\partial q_{ij} = p_{ij}(1 - \eta_{ij})$ gives the weight of the division in i's ranking. The induced game is then an asymmetric roommate problem, for which a stable matching need not exist (Gale and Shapley, 1962). For an explicit instance in the electorally induced domain, let the votes be random, placing probability $\frac{1}{3}$ on each of three profiles $v^{(1)}, v^{(2)}, v^{(3)}$ under which the bloc's marginal-average order is $a_1 > a_2 > a_3$,

$a_2 > a_3 > a_1$, and $a_3 > a_1 > a_2$ respectively, and under each of which the pooled pair wins one seat; any strict order of averages is induced by some votes, so the three profiles exist. Then $q_{12} = \Pr[a_1 > a_2] = \frac{2}{3}$, $q_{23} = \frac{2}{3}$, and $q_{31} = \frac{2}{3}$, a cyclic recipiency. With $\eta = 0$ and common win probability p, $u_i(j) = p\, q_{ij}$, so

$$u_1(2) = \tfrac{2}{3}p > u_1(3) = \tfrac{1}{3}p, \quad u_2(3) = \tfrac{2}{3}p > u_2(1) = \tfrac{1}{3}p, \quad u_3(1) = \tfrac{2}{3}p > u_3(2) = \tfrac{1}{3}p,$$

a preference cycle $1 \to 2 \to 3 \to 1$. Each of the three matchings leaves one list unmatched and is blocked ($\{1,2\}$ by $\{2,3\}$, $\{2,3\}$ by $\{1,3\}$, and $\{1,3\}$ by $\{1,2\}$), and the all-unmatched profile by any pair. Hence no stable matching exists, and the cyclic recipiency is generated by an explicit distribution over vote profiles. □

Proof of Proposition 4. Process the admissible pairs in $\succ$-order, adding a pair to M when both endpoints are unmatched. Suppose $\{i,j\}$ blocks the result, so $u_i(j) > u_i(M)$ and $u_j(i) > u_j(M)$. By the common-order hypothesis $\{i,j\}$ $\succ$-precedes i's and j's edges in M (with an unmatched list treated as $\succ$-dominated by any admissible edge). Hence when $\{i,j\}$ was processed, before those edges, both i and j were still unmatched, and the procedure would have added $\{i,j\}$, a contradiction. So M is stable. Setting $\succ$ to the order by p recovers Theorem 1(ii); the profile of Theorem 1(iii) induces the cycle $1 \to 2 \to 3 \to 1$, which no linear order $\succ$ can generate. □

Proof of Proposition 5. Under separability (Assumption 1) the win and recipiency probabilities do not vary with the matching, so each list i has the fixed utility $u_i(j) = p_{ij}[q_{ij} + \eta_{ij}(1 - q_{ij})]$ over its admissible partners and value 0 from remaining unmatched. For generic votes and cohesion these utilities are distinct, so the induced preferences are strict; a partner j is acceptable to i whenever $p_{ij} > 0$, and every acceptable partner is preferred to being unmatched. This is a roommate problem on G. Tan (1991) shows that every roommate problem has a stable partition, that its odd rings are the same in every stable partition, and that a stable matching exists if and only if that stable partition has no odd ring; we read the characterisation off this induced problem.

(i) A common order forbids rings. Suppose the lists share the order $\succ$ of Proposition 4, and suppose for contradiction that $(x_1, \ldots, x_k)$ is a ring. Writing $e_t = \{x_t, x_{t+1}\}$ for the successor edge, the ring condition $u_{x_t}(x_{t+1}) > u_{x_t}(x_{t-1})$ reads, under the common order, as $e_t \succ e_{t-1}$ for every t modulo k. Chaining these around the cycle gives $e_1 \succ e_k \succ e_{k-1} \succ \cdots \succ e_1$, which is impossible. So no ring of any length exists, in particular no odd one, and a stable matching exists. Full cohesion is the case $\succ =$ the order by p of Theorem 1(ii).

(ii) The minimal odd ring. A ring has at least three lists, so the smallest odd ring has exactly three. The profile of Theorem 1(iii), with $\eta = 0$, common win probability $p > 0$, and the cyclic recipiency $q_{12} = q_{23} = q_{31} = \frac{2}{3}$, gives $u_i(\text{successor}) = \frac{2}{3}p$ and $u_i(\text{predecessor}) = \frac{1}{3}p$

along $1 \to 2 \to 3 \to 1$, so $\{1,2,3\}$ is a ring, and an odd one. The three lists form a single ring, which is their stable partition, so by Tan's condition no stable matching exists; the direct check in the proof of Theorem 1(iii), where each of the three pairings is blocked and the empty matching by any pair, confirms it. No odd ring is smaller, so this is the minimal obstruction. □

Proof of Corollary 1. Take admissible pairs with common values $p_{ab} = p_{cd} = x$ and $p_{bc} = y$ with $y > x$, and no other admissible edges among a, b, c, d. The greedy procedure of Theorem 1(ii) first adds $\{b, c\}$ and leaves a, d unmatched, for total y; this matching is stable. The matching $\{a, b\}, \{c, d\}$ has total $2x$, which exceeds y whenever $x > y/2$, yet it is not stable because $\{b, c\}$ blocks it. Hence the stable matching is not efficient. □

Proof of Proposition 6. The value of i's realised pair is $u_i(M)$ by definition; if $\{i, j\} = M(i)$ is a signed pair then $u_i(j) = u_i(M)$ identically, and $u_i(j) = p_{ij}[q_{ij} + \eta_{ij}(1 - q_{ij})] \geq 0$ for every $\eta_{ij} \geq 0$, so signing yields no constraint beyond non-negativity. Pairwise stability is the statement that no admissible unsigned pair blocks, that is, for each such $\{i, j\}$ it is not the case that both $u_i(j) > u_i(M)$ and $u_j(i) > u_j(M)$; negating gives $u_i(j) \leq u_i(M)$ or $u_j(i) \leq u_j(M)$. Each side is a half-space in the cohesion weights, through $u_i(j) = p_{ij}[q_{ij} + \eta_{ij}(1 - q_{ij})]$ and the analogous $u_i(M)$, so each unsigned pair contributes a union of two half-spaces, and the identified set is their intersection, a union of regions. □

Proof of Proposition 7. Transferring one seat from ℓ to r changes their deviations to $e_\ell - \delta$ and $e_r + \delta$ and leaves the others fixed, so the sum of squared deviations changes by $(e_\ell - \delta)^2 + (e_r + \delta)^2 - e_\ell^2 - e_r^2 = 2\delta(e_r - e_\ell) + 2\delta^2 = 2\delta(e_r - e_\ell + \delta)$. A least-squares index is increasing in this sum, so it falls iff $e_r - e_\ell + \delta < 0$, that is $e_\ell - e_r > \delta$. □

References

Aumann, R. J., and M. Maschler (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory* 36(2), 195–213.

Balinski, M. L., and H. P. Young (2001). *Fair Representation: Meeting the Ideal of One Man, One Vote* (2nd ed.). Washington, DC: Brookings Institution Press.

Blais, A., and I. H. Indridason (2007). Making candidates count: the logic of electoral alliances in two-round legislative elections. *Journal of Politics* 69(1), 193–205.

Bochsler, D. (2010). Who gains from apparentments under D'Hondt? *Electoral Studies* 29(4), 617–627.

Brams, S. J., and D. M. Kilgour (2001). Competitive fair division. *Journal of Political Economy* 109(2), 418–443.

Brams, S. J., D. M. Kilgour, and C. Klamler (2012). The undercut procedure: an algorithm for the envy-free division of indivisible items. *Social Choice and Welfare* 39(2), 615–631.

Chung, K. S. (2000). On the existence of stable roommate matchings. *Games and Economic Behavior* 33(2), 206–230.

Dewan, T. (2026). Coalition Division as a Contested Garment. Working paper, SSRN 7293718.

Gale, D., and L. S. Shapley (1962). College admissions and the stability of marriage. *American Mathematical Monthly* 69(1), 9–15.

Golder, S. N. (2006). Pre-electoral coalition formation in parliamentary democracies. *British Journal of Political Science* 36(2), 193–212.

Hortalà-Vallvé, R., J. Meriläinen, and J. Tukiainen (2024). Pre-electoral coalitions and the distribution of political power. *Public Choice* 198, 47–67.

Irving, R. W. (1985). An efficient algorithm for the stable roommates problem. *Journal of Algorithms* 6(4), 577–595.

Karpov, A. (2014). Alliance incentives under the d'Hondt method. *Mathematical Social Sciences* 74, 1–7.

Lee, M., R. D. McKelvey, and H. Rosenthal (1979). Game theory and the French alliances of 1951. *International Journal of Game Theory* 8(1), 27–53.

Leutgäb, P., and F. Pukelsheim (2009). List apparentments in local elections: a lottery. *Homo Oeconomicus* 26(3/4), 489–500.

O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences* 2(4), 345–371.

Pérez, J., and O. De la Cruz (2014). Implementation of the Jefferson–d'Hondt rule in the formation of a parliamentary committee. *Social Choice and Welfare* 42(1), 17–30.

Pukelsheim, F. (2014). *Proportional Representation: Apportionment Methods and Their Applications*. Cham: Springer.

Roth, A. E., and M. A. O. Sotomayor (1990). *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge: Cambridge University Press.

Sasaki, H., and M. Toda (1996). Two-sided matching problems with externalities. *Journal of Economic Theory* 70(1), 93–108.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics* 17(6), 1163–1170.

Schuster, K., F. Pukelsheim, M. Drton, and N. R. Draper (2003). Seat biases of apportionment methods for proportional representation. *Electoral Studies* 22(4), 651–676.

Tan, J. J. M. (1991). A necessary and sufficient condition for the existence of a complete stable matching. *Journal of Algorithms* 12(1), 154–178.

Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. *Mathematical Social Sciences* 45(3), 249–297.

Thomson, W. (2015). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: an update. *Mathematical Social Sciences* 74, 41–59.